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Soft Union-Difference Product of Groups

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Abstract

Soft set theory constitutes a highly flexible and mathematically rigorous framework for modeling and analyzing real-world phenomena characterized by uncertainty, ambiguity, and parameter-dependent variability—features that frequently arise in disciplines such as decision sciences, engineering, economics, and information systems. Central to this theoretical apparatus are the fundamental operations and product constructions on soft sets, which collectively give rise to a rich and expressive algebraic infrastructure capable of accommodating complex parametric interdependencies. In this study, we introduce a novel product, termed the soft union–difference product, specifically defined for soft sets whose parameter sets possess a group structure. A thorough axiomatic and structural analysis of this is conducted, with special attention to its algebraic compatibility with generalized notions of soft subsethood and soft equality. Through this analysis, we uncover the product's intrinsic structural properties and demonstrate its capacity to preserve essential algebraic features within group-parameterized soft set systems. Furthermore, we conduct a comprehensive algebraic investigation of the soft union–difference product, examining its closure, associativity, idempotency, commutativity, absorbing property, and distributivity, as well as its interaction with other established soft products defined on groups and null soft sets. These investigations reveal two pivotal theoretical implications: first, they reinforce the internal algebraic coherence of soft set theory by situating the newly defined product within a formally consistent operational framework; second, they lay a conceptual foundation for the emergence of a soft group theory that structurally parallels classical group-theoretic constructions. Given that the advancement of soft algebraic systems is inherently predicated on rigorously defined operations and systematically articulated product frameworks, the present study makes a substantial contribution to the formal algebraic refinement and theoretical evolution of soft set theory. Beyond their theoretical merit, the proposed constructions also offer concrete methodological tools for the development of group-based soft computational models, with potential applications in multi-criteria decision-making, uncertainty-aware classification systems, and data-driven analysis under parameter uncertainty.

Keywords: *Soft Sets; Soft Subsets; Soft Equalities; Soft Union-Difference Product.*

INTRODUCTION

A multitude of advanced mathematical paradigms have been developed to model and analyze phenomena characterized by intrinsic uncertainty, imprecision, and vagueness—conditions pervasive across diverse domains such as engineering, economics, the social sciences, and healthcare. Despite their methodological sophistication, many of these frameworks suffer from fundamental structural and epistemological limitations, as thoroughly critiqued in Molodtsov's seminal work (1999). For instance, fuzzy set theory, introduced by Zadeh (1965), is inherently constrained by the subjectivity embedded in membership function selection, whereas probabilistic models rely on idealized assumptions of repeatability and known distributions, thereby restricting their efficacy in non-replicable or epistemically sparse environments. To address these foundational shortcomings, Molodtsov (1999) introduced soft set theory as a structurally pliant and conceptually robust alternative for modeling parameter-dependent uncertainty. Soft set theory eliminates the need for prerequisite axioms such as exact membership gradation or probability distributions, thereby facilitating broader applicability in decision theory, optimization, information systems, and game theory.

Since its inception, the formal architecture of soft set theory has evolved significantly. The axiomatic groundwork—initially laid by Maji et al. (2003) through the introduction of soft

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subsets, equality, and elementary operations such as union, intersection, and AND/OR-products—was subsequently refined by Pei and Miao (2005), who emphasized information-theoretic perspectives and reformulated foundational operations to accommodate relational structures. Further operational enrichment was realized through the work of Ali et al. (2009), who introduced extended and restricted variants of classical operations, thereby enhancing the structural expressiveness of soft sets. Parallel developments by Yang (2008), Feng et al. (2010), Jiang et al. (2010), Ali et al. (2011), Neog and Sut (2011), Fu (2011), Ge and Yang (2011), Singh and Onyeozili (2012a, 2012b, 2012c, 2012d), Zhu and Wen (2013), Onyeozili and Gwary (2014), and Sen (2014), pivoted toward a deeper algebraic formalization of soft set operations. These studies addressed interpretational ambiguities, proposed refined operation models, and laid the theoretical groundwork for the emergence of soft algebraic systems. Recent advances have further expanded the algebraic foundations of soft set theory. Notable contributions include those by Eren and Çalışıcı (2019), Stojanović (2021), Sezgin et al. (2023a, 2023b), Sezgin and Aybek (2023), Sezgin and Dağtoros (2023), Sezgin and Demirci (2023), Sezgin and Çalışıcı (2024), Sezgin and Yavuz (2023a, 2023b; 2024), Sezgin and Çağman (2024,2025), Sezgin and Sarıalioğlu (2024a, 2024b), and Sezgin and Şenyiğit (2025) who have proposed a broad class of novel operations, each subjected to rigorous algebraic scrutiny.

A central axis of this advancement is the refinement and generalization of soft equality and subsethood concepts. The original formulation of soft subsets by Maji et al. (2003) was extended by Pei and Miao (2005) and Feng et al. (2010), while Qin and Hong (2010) introduced soft congruences, extending the formalism to accommodate refined equivalence structures. Jun and Yang (2011) further generalized these constructs by defining J-soft equality and establishing new distributive frameworks. In continuation, Liu et al. (2012) developed L-soft subsets and L-equalities, revealing nontrivial structural divergences and exposing the non-universality of classical distributive laws within enriched soft frameworks. Extending this trajectory, Feng and Li (2013) delivered a comprehensive taxonomy of soft subsets and rigorously examined the algebraic properties of AND- and OR-products, especially under the L-soft framework. Their results resolved key issues of associativity, commutativity, and distributivity, demonstrating that L-equalities induce congruence structures within free soft algebras, wherein the corresponding quotient algebras assume the form of commutative semigroups. Building on this refinement, Sezgin et al. (2025a) conducted an exhaustive algebraic investigation of the AND-product under varying soft equality regimes—specifically L-, J-, and M-equalities—along with an analysis of soft F-subsets. Their work systematically verified the algebraic properties of these operations, including associativity, commutativity, and idempotency, under multiple relational conditions. Generalized equality types such as g-soft, gf-soft, and T-soft equalities have also been explored, alongside relaxations of parameter constraints and lattice-theoretic extensions by Abbas et al. (2014, 2017), Al-shami (2019), and Al-shami and El-Shafei (2020).

To strengthen the operational clarity and broaden applicability, Çağman and Enginoğlu (2010) redefined the core operations introduced by Maji et al. (2003), offering a functionally coherent and algebraically rigorous formalism. Simultaneously, the study of soft product structures has gained considerable traction. Distinct variants of the soft intersection-union product have been developed for rings (Sezer, 2012), semigroups (Sezgin, 2016), and groups (Muştuoğlu et al., 2016), resulting in the construction of soft union rings, semigroups, and groups, respectively. In a parallel line of inquiry, the soft union-intersection product has been formalized for groups (Kaygısız, 2012), semigroups (Sezer et al., 2015), and rings (Sezgin et al., 2017), resulting in the construction of soft union groups, semigroups, and rings, respectively, with the resulting algebraic structure contingent upon the nature of identity and inverse elements within the parameter domain.

Motivated by these developments, the present study introduces a novel product on soft sets—termed the "soft union-difference product"—defined over a group-theoretic parameter. We conduct a rigorous and comprehensive algebraic analysis of this product, with particular emphasis on its interaction with various soft subset classes and generalized notions of soft equality. Additionally, the proposed operation is systematically compared with previously introduced soft products within the framework of soft subset taxonomy, offering a deeper theoretical understanding of their respective representational efficacy and algebraic compatibility. Moreover, a rigorous structural analysis is performed to examine the interaction between the proposed product and the null soft set, further elucidating its foundational algebraic behavior. Our findings reveal that the proposed product not only possesses desirable internal coherence but also extends the expressive capacity of soft algebraic systems, permitting the generalization of classical algebraic structures and providing new tools for resolving longstanding theoretical problems. These results significantly augment the foundational algebra of soft set theory and lay the conceptual groundwork for the formal emergence of a novel branch: "soft group theory", constructed around the proposed binary operation. The remainder of the manuscript is structured as follows: Section 2 surveys the foundational definitions and prior developments in soft set theory essential to our analysis. Section 3 introduces the soft union-difference product and presents a detailed algebraic treatment of its structural properties and relational behaviors. Finally, Section 5 consolidates the main findings and delineates future research trajectories aimed at deepening the algebraic landscape of soft set theory.

PRELIMINARIES

This section undertakes a meticulous and systematic reassessment of the fundamental definitions and algebraic scaffolding that constitute the formal basis for the theoretical constructs developed in the subsequent sections. Although the notion of soft sets was originally formulated by Molodtsov (1999), the definitional architecture and associated operational mechanics underwent substantial refinement in the seminal work of Çağman and Enginoğlu (2010), aimed at enhancing both the axiomatic rigor and the applicability of the theory within broader analytical contexts. The present study adopts this revised formulation as its foundational paradigm. Consequently, all ensuing algebraic investigations, structural formulations, and theoretical extrapolations are rigorously situated within the framework of this refined conceptual apparatus.

Definition 2.1. (Çağman and Enginoğlu, 2010) Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set $f_{\mathcal{H}}$ over U is a function such that $f_{\mathcal{H}}: E \to P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)): w \in E\}$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2.2. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null $\mathcal{SS}$ and indicated by $\emptyset_E$, and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute $\mathcal{SS}$ and indicated by U_E.

Definition 2.3. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. If $f_{\mathcal{H}}(w) \subseteq g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is a soft subset of $g_{\aleph}$ and indicated by $f_{\mathcal{H}} \tilde{\subseteq} g_{\aleph}$. If $f_{\mathcal{H}}(w) = g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is called soft equal to $g_{\aleph}$, and denoted by $f_{\mathcal{H}} = g_{\aleph}$.

Definition 2.4. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $f_{\mathcal{H}}$ denoted by $f_{\mathcal{H}}{}^{c}$, is defined by the soft set $f_{\mathcal{H}}{}^{c}: E \to P(U)$ such that $f_{\mathcal{H}}{}^{c}(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.5. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, the union of $f_{\mathcal{H}}$ and $g_{\aleph}$ is the $\mathcal{SS}$ $f_{\mathcal{H}} \tilde{\cup} g_{\aleph}$, where $(f_{\mathcal{H}} \tilde{\cup} g_{\aleph})(w) = f_{\mathcal{H}}(w) \cup g_{\aleph}(w)$, for all $w \in E$.

Definition 2.6. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_S g_{\aleph}$, if for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two $\mathcal{SS}$s f_K and $g_{\aleph}$ are said to be soft S-equal, denoted by $f_K =_S g_{\aleph}$, if $f_K \tilde{\subseteq}_S g_{\aleph}$ and $g_{\aleph} \tilde{\subseteq}_S f_K$.

It is obvious that if $f_K =_S g_{\aleph}$, then f_K and $g_{\aleph}$ are the same constant functions, that is, for all $w \in E$, $f_K(w) = g_{\aleph}(w) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set.

Definition 2.7. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft A-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_A g_{\aleph}$, if, for each $\rho, \tau \in E$, $f_K(\rho) \subseteq g_{\aleph}(\tau)$.

Definition 2.8. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-complement of $g_{\aleph}$, denoted by $f_K =_S (g_{\aleph})^c$, if, for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = U \setminus \mathcal{D}$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an $\mathcal{SS}$ parameterized by G.

Definition 2.9. (Ay and Sezgin, 2025) Let f_G and g_G be two $\mathcal{SS}$s over U. Then, the soft intersection-plus product $f_G \otimes_{i/p} g_G$ is defined by

$$(f_G \otimes_{i/p} g_G)(x) = \bigcap_{x=yz} (f_G(y) + g_G(z)) = \bigcap_{x=yz} \left((f_G(y))' \cup g_G(z)\right), \qquad y, z \in G$$

for all $x \in G$.

For more on more on plus (+) operation of sets, we refer to Sezgin et al. (2023c), more on $\mathcal{SS}$s, we refer to Aktas and Çağman (2007), Alcantud et al. (2024), Ali et al. (2015), Ali et al. (2022), Atagün et al. (2019), Atagün and Sezgin (2015), Atagün and Sezer (2015), Atagün and Sezgin (2017), Atagün and Sezgin (2018), Atagün and Sezgin (2022), Feng et al. (2008), Gulistan and Shahzad (2014), Gulistan et al. (2018); Jana et al. (2019), Karaaslan (2019), Khan et al. (2017), Mahmood et al. (2015), Mahmood et al. (2018), Manikantan et al. (2023), Memiş (2022), Özlü and Sezgin (2020), Riaz et al. (2023), Sezer and Atagün (2016), Sezer et al. (2017), Sezer et al. (2013), Sezer et al. (2014), Sezgin et al. (2019a, 2019b), Sezgin and İlgin (2024a, 2024b), Sezgin et al. (2022), Sezgin and Onur (2024), Sezgin et al. (2024a,2024b), Sezgin and Orbay (2022), Sun et al. (2008), Tunçay and Sezgin (2016), Ullah et al. (2018).

SOFT UNION-DIFFERENCE PRODUCT OF GROUPS

In this section, we introduce a novel product of $\mathcal{SS}$s, designated as the "soft union-difference product", wherein the parameter set is structured as a group. A comprehensive algebraic examination of the proposed product is undertaken, with the analysis oriented toward a rigorous characterization of its intrinsic structural properties. Particular emphasis is placed on the product's interaction with various generalized notions of soft equality and the stratification of soft subsets under distinct inclusion criteria. To concretize the abstract formulations and illuminate key algebraic phenomena, the theoretical discourse is supplemented by a series of representative and analytically instructive examples. Moreover, we investigate the algebraic properties of the operation including closure, associativity, commutativity, idempotency, absorbing element, right-distributive properties of the soft union-difference product over the union operation of $\mathcal{SS}$s, as well as its interactions with null and absolute soft set, thereby delineating its compatibility with the established algebraic operations and further situating the product within the broader algebraic hierarchy of $\mathcal{SS}$ theory.

Definition 3.1. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Then, the soft union-difference product $\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G$ is defined by

$$(\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G)(x) = \bigcup_{x=yz} \left(\mathscr{f}_G(y) \setminus \mathscr{g}_G(z)\right), \qquad y, z \in G$$

for all $x \in G$.

Note here that since G is a group, there always exist $y, z \in G$ such that $x = yz$, for all $x \in G$. Let the order of the group G be n, that is, $|G| = n$. Then, it is obvious that there exist n distinct representations for each $x \in G$ such that $x = yz$, where $y, z \in G$.

Note 3.2. The soft union-difference product is well-defined in $S_G(U)$. In fact, let $\mathscr{f}_G, \mathscr{g}_G, \mathscr{m}_G, \mathscr{k}_G \in S_G(U)$ such that $(\mathscr{f}_G, \mathscr{g}_G) = (\mathscr{m}_G, \mathscr{k}_G)$. Then, $\mathscr{f}_G = \mathscr{m}_G$ and $\mathscr{g}_G = \mathscr{k}_G$, implying that $\mathscr{f}_G(x) = \mathscr{m}_G(x)$ and $\mathscr{g}_G(x) = \mathscr{k}_G(x)$, for all $x \in G$. Thereby, for all $x \in G$,

$$\begin{aligned}(\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G)(x) &= \bigcup_{x=yz} \left(\mathscr{f}_G(y) \setminus \mathscr{g}_G(z)\right) \\ &= \bigcup_{x=yz} \left(\mathscr{m}_G(y) \setminus \mathscr{k}_G(z)\right) \\ &= (\mathscr{m}_G \otimes_{u/d} \mathscr{k}_G)(x)\end{aligned}$$

Hence, $\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G = \mathscr{m}_G \otimes_{u/d} \mathscr{k}_G$.

Example 3.3. Consider the group $G = \{\rho, \tau\}$ with the following operation:

$\cdot$	ρ	τ
ρ	τ	ρ
τ	ρ	τ

Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s over $U = D_2 = \{< x, y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$$\mathscr{f}_G = \{(\rho, \{e, x, y\}), (\tau, \{x, yx\})\} \text{ and } \mathscr{g}_G = \{(\rho, \{e, y\}), (\tau, \{e, yx\})\}$$

Since $\rho = \rho\tau = \tau\rho$, $(\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G)(\rho) = \left(\mathscr{f}_G(\rho) \setminus \mathscr{g}_G(\tau)\right) \cup \left(\mathscr{f}_G(\tau) \setminus \mathscr{g}_G(\rho)\right) = \{x, y, yx\}$, and since $\tau = \rho\rho = \tau\tau$, $(\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G)(\tau) = \left(\mathscr{f}_G(\rho) \setminus \mathscr{g}_G(\rho)\right) \cup \left(\mathscr{f}_G(\tau) \setminus \mathscr{g}_G(\tau)\right) = \{x\}$ is obtained. Hence,

$$f_G \otimes_{u/d} g_G = \{(\rho, \{x, y, yx\}), (\tau, \{x\})\}$$

Proposition 3.4. The set $S_G(U)$ is closed under the soft union-difference product. That is, if f_G and g_G are two SSs, then so is $f_G \otimes_{u/d} g_G$.

PROOF. It is obvious that the soft union-difference product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft union-difference product. □

Proposition 3.5. The soft union-difference product is not associative in $S_G(U)$.

PROOF. Consider the group G and the SSs f_G and g_G in Example 3.3. Let h_G be an SS over $U = \{e, x, y, yx\}$ such that $h_G = \{(\rho, \{y, yx\}), (\tau, \{e, x, y\})\}$.

Since $f_G \otimes_{u/d} g_G = \{(\rho, \{x, y, yx\}), (\tau, \{x\})\}$, then

$$(f_G \otimes_{u/d} g_G) \otimes_{u/d} h_G = \{(\rho, \{x, yx\}), (\tau, \{x\})\}$$

Moreover, since $g_G \otimes_{u/d} h_G = \{(\rho, \{e\}), (\tau, \{e, yx\})\}$, then

$$f_G \otimes_{u/d} (g_G \otimes_{u/d} h_G) = \{(\rho, \{x, y, yx\}), (\tau, \{x, y\})\}$$

Thereby, $(f_G \otimes_{u/d} g_G) \otimes_{u/d} h_G \neq f_G \otimes_{u/d} (g_G \otimes_{u/d} h_G)$. □

Proposition 3.6. The soft union-difference product is not commutative in $S_G(U)$.

PROOF. Consider the SSs f_G and g_G in Example 3.3. Then,

$$f_G \otimes_{u/d} g_G = \{(\rho, \{x, y, yx\}), (\tau, \{x\})\} \text{ and } g_G \otimes_{u/d} f_G = \{(\rho, \{e, y, yx\}), (\tau, \{e\})\}$$

implying that $f_G \otimes_{u/d} g_G \neq g_G \otimes_{u/d} f_G$. □

Proposition 3.7. The soft union-difference product is not idempotent in $S_G(U)$.

PROOF. Consider the SS f_G in Example 3.3. Then,

$$f_G \otimes_{u/d} f_G = \{(\rho, \{e, y, yx\}), (\tau, \emptyset)\}$$

implying that $f_G \otimes_{u/d} f_G \neq f_G$. □

Proposition 3.8. Let f_G be a constant SS. Then, $f_G \otimes_{u/d} f_G = \emptyset_G$.

PROOF. Let f_G be a constant SS such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Then, for all $x \in G$,

$$(f_G \otimes_{u/d} f_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus f_G(z)) = \emptyset_G(x)$$

Thereby, $f_G \otimes_{u/d} f_G = \emptyset_G$. □

Remark 3.9. Let $S_G^*(U)$ be the collection of all constant $\mathcal{SS}$. Then, the soft union-difference product is not idempotent in $S_G^*(U)$ either.

Proposition 3.10. $\emptyset_G$ is the left absorbing element of the soft union-difference product in $S_G(U)$.

PROOF. Let $x \in G$. Then, for all $x \in G$,

$$\begin{aligned}(\emptyset_G \otimes_{u/d} f_G)(x) &= \bigcup_{x=yz} (\emptyset_G(y) \setminus f_G(z)) \\ &= \bigcup_{x=yz} (\emptyset \setminus f_G(z)) \\ &= \emptyset_G(x)\end{aligned}$$

Thus, $\emptyset_G \otimes_{u/d} f_G = \emptyset_G$. □

Proposition 3.11. $\emptyset_G$ is not the right absorbing element of the soft union-difference product in $S_G(U)$.

PROOF. Consider the $\mathcal{SS}$ f_G in Example 3.3. Then,

$$f_G \otimes_{u/d} \emptyset_G = \{(\rho, U), (\tau, U)\}$$

implying that $f_G \otimes_{u/d} \emptyset_G \neq \emptyset_G$. □

Remark 3.12. $\emptyset_G$ is not the absorbing element of the soft union-difference product in $S_G(U)$.

Proposition 3.13. Let f_G be a constant $\mathcal{SS}$. Then, $f_G \otimes_{u/d} \emptyset_G = f_G$.

PROOF. Let f_G be a constant $\mathcal{SS}$ such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Then, for all $x \in G$,

$$(f_G \otimes_{u/d} \emptyset_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus \emptyset_G(z)) = \bigcup_{x=yz} (f_G(y) \setminus \emptyset) = f_G(x)$$

Thereby, $f_G \otimes_{u/d} \emptyset_G = f_G$. □

Remark 3.14. $\emptyset_G$ is not the absorbing element of the soft union-difference product in $S_G^*(U)$. Moreover, $\emptyset_G$ is the right identity element of the soft union-difference product in $S_G^*(U)$.

Proposition 3.15. Let f_G be an $\mathcal{SS}$. Then, $f_G \otimes_{u/d} U_G = \emptyset_G$.

PROOF. Let f_G be a $\mathcal{SS}$. Then, for all $x \in G$,

$$(f_G \otimes_{u/d} U_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus U_G(z)) = \bigcup_{x=yz} (f_G(y) \setminus U) = \emptyset_G(x)$$

Thereby, $f_G \otimes_{u/d} U_G = \emptyset_G$. □

Proposition 3.16. Let f_G be a constant $\mathcal{SS}$. Then, $U_G \otimes_{u/d} f_G = f_G{}^c$.

PROOF. Let f_G be a constant $\mathcal{SS}$ such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Then, for all $x \in G$,

$$(U_G \otimes_{u/d} f_G)(x) = \bigcup_{x=yz} (U_G(y) \setminus f_G(z)) = \bigcup_{x=yz} (U \setminus f_G(z)) = f_G{}^c(x)$$

Thereby, $U_G \otimes_{u/d} f_G = f_G{}^c$. □

Proposition 3.17. Let f_G and g_G be two SSs. Then, $(f_G \otimes_{u/d} g_G)^c = f_G \otimes_{i/p} g_G$.

PROOF. Let f_G and g_G be two SSs. Then, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/d} g_G)^c(x) &= \left(\bigcup_{x=yz} (f_G(y) \setminus g_G(z))\right)' \\ &= \bigcap_{x=yz} (f_G(y) \setminus g_G(z))' \\ &= \bigcap_{x=yz} (f_G(y) \cap g_G{}^c(z))' \\ &= \bigcap_{x=yz} (f_G{}^c(y) \cup g_G(z)) \\ &= (f_G \otimes_{i/p} g_G)(x)\end{aligned}$$

Thereby, $(f_G \otimes_{u/d} g_G)^c = f_G \otimes_{i/p} g_G$. □

Theorem 3.18. Let f_G and g_G be two SSs. Then, $f_G \otimes_{u/d} g_G = \emptyset_G$ if only if $f_G \,\tilde{\subseteq}_A\, g_G$.

PROOF. Suppose that $f_G \,\tilde{\subseteq}_A\, g_G$. Then, $f_G(\rho) \subseteq g_G(\tau)$, for each $\rho, \tau \in G$. Thus, for all $x \in G$,

$$(f_G \otimes_{u/d} g_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus g_G(z)) = \emptyset = \emptyset_G(x)$$

Thereby, $f_G \otimes_{u/d} g_G = \emptyset_G$.

Conversely, suppose that $f_G \otimes_{u/d} g_G = \emptyset_G$. That is, $(f_G \otimes_{u/d} g_G)(x) = \emptyset_G(x)$, for each $x \in G$. Then, for all $x \in G$,

$$\emptyset_G(x) = (f_G \otimes_{u/d} g_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus g_G(z)) = \emptyset$$

This implies that $f_G(y) \setminus g_G(z) = \emptyset$, for all $y, z \in G$. Thus, $f_G \,\tilde{\subseteq}_A\, g_G$. □

Proposition 3.19. Let f_G and g_G be two SSs such that $f_G \,\tilde{\subseteq}_S\, (g_G)^c$. Then, $f_G \otimes_{u/d} g_G = f_G$.

PROOF. Let f_G and g_G be two SSs and $f_G \,\tilde{\subseteq}_S\, (g_G)^c$. Then, for all $x \in G$, $f_G(x) = A$, $g_G(x) = B$, where A and B are two fixed sets and $A \subseteq B'$. Then, for all $x \in G$,

$$(f_G \otimes_{u/d} g_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus g_G(z)) = f_G(x)$$

Thereby, $f_G \otimes_{u/d} g_G = f_G$. □

Proposition 3.20. Let f_G and g_G be two SSs such that $(g_G)^c \,\tilde{\subseteq}_S\, f_G$. Then, $f_G \otimes_{u/d} g_G = g_G{}^c$.

PROOF. Let f_G and g_G be two SSs and $(g_G)^c \,\tilde{\subseteq}_S\, f_G$. Then, for all $x \in G$, $f_G(x) = A$, $g_G(x) = B$, where A and B are two fixed sets and $B' \subseteq A$. Moreover, since $B' \subseteq A$, for all $x \in G$,

$$(f_G \otimes_{u/d} g_G)(x) = \bigcup_{x=yz} (f_G(y) \setminus g_G(z)) = g_G{}^c(x)$$

Thereby, $f_G \otimes_{u/d} g_G = g_G{}^c$. □

Proposition 3.21. Let f_G, g_G, and h_G be three SSs. If $f_G \tilde{\subseteq} g_G$, then $f_G \otimes_{u/d} h_G \tilde{\subseteq} g_G \otimes_{u/d} h_G$ and $h_G \otimes_{u/d} g_G \tilde{\subseteq} h_G \otimes_{u/d} f_G$.

PROOF. Let f_G, g_G, and h_G be three SSs such that $f_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $f_G(x) \subseteq g_G(x)$. Thus, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{u/d} h_G)(x) &= \bigcup_{x=yz} \big(f_G(y) \setminus h_G(z)\big) \\ &\subseteq \bigcup_{x=yz} \big(g_G(y) \setminus h_G(z)\big) \\ &= (g_G \otimes_{u/d} h_G)(x)\end{aligned}$$

for all $x \in G$, implying that $f_G \otimes_{u/d} h_G \tilde{\subseteq} g_G \otimes_{u/d} h_G$. Moreover, since $(g_G(x))' \subseteq (f_G(x))'$, for all $x \in G$,

$$\begin{aligned}(h_G \otimes_{u/d} g_G)(x) &= \bigcup_{x=yz} \big(h_G(y) \setminus g_G(z)\big) \\ &\subseteq \bigcup_{x=yz} \big(h_G(y) \setminus f_G(z)\big) \\ &= (h_G \otimes_{u/d} f_G)(x)\end{aligned}$$

implying that $h_G \otimes_{u/d} g_G \tilde{\subseteq} h_G \otimes_{u/d} f_G$. □

Proposition 3.22. Let f_G, g_G, m_G and k_G be four SSs. If $m_G \tilde{\subseteq} f_G$ and $k_G \tilde{\subseteq} g_G$, then $m_G \otimes_{u/d} g_G \tilde{\subseteq} f_G \otimes_{u/d} k_G$ and $k_G \otimes_{u/d} f_G \tilde{\subseteq} g_G \otimes_{u/d} m_G$.

PROOF. Let f_G, g_G, m_G, and k_G be four SSs such that $m_G \tilde{\subseteq} f_G$ and $k_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $m_G(x) \subseteq f_G(x)$ and $k_G(x) \subseteq g_G(x)$. Thus, for all $x \in G$, $(g_G(x))' \subseteq (k_G(x))'$ and $(f_G(x))' \subseteq (m_G(x))'$ for all $x \in G$. Thereby, for all $x \in G$,

$$\begin{aligned}(m_G \otimes_{u/d} g_G)(x) &= \bigcup_{x=yz} \big(m_G(y) \setminus g_G(z)\big) \\ &\subseteq \bigcup_{x=yz} \big(f_G(y) \setminus k_G(z)\big) \\ &= (f_G \otimes_{u/d} k_G)(x)\end{aligned}$$

for all $x \in G$, implying that $m_G \otimes_{u/d} g_G \tilde{\subseteq} f_G \otimes_{u/d} k_G$. Similarly, for all $x \in G$,

$$\begin{aligned}(k_G \otimes_{u/d} f_G)(x) &= \bigcup_{x=yz} \big(k_G(y) \setminus f_G(z)\big) \\ &\subseteq \bigcup_{x=yz} \big(g_G(y) \setminus m_G(z)\big) \\ &= (g_G \otimes_{u/d} m_G)(x)\end{aligned}$$

is obtained. Thereby, $k_G \otimes_{u/d} f_G \tilde{\subseteq} g_G \otimes_{u/d} m_G$. □

Proposition 3.23. The soft union-difference product distributes over the union operation of SSs from the right side.

PROOF. Let f_G, g_G, and h_G be three SSs. Then, for all $x \in G$,

$$\begin{aligned}\big((f_G \tilde{\cup} g_G) \otimes_{u/d} h_G\big)(x) &= \bigcup_{x=yz} \big((f_G \tilde{\cup} g_G)(y) \setminus h_G(z)\big) \\ &= \bigcup_{x=yz} \Big(\big(f_G(y) \cup g_G(y)\big) \setminus h_G(z)\Big) \\ &= \bigcup_{x=yz} \Big(\big(f_G(y) \setminus h_G(z)\big) \cup \big(g_G(y) \setminus h_G(z)\big)\Big)\end{aligned}$$

$$= \left[\bigcup_{x=yz}\left(f_G(y) \setminus h_G(z)\right)\right] \cup \left[\bigcup_{x=yz}\left(g_G(y) \setminus h_G(z)\right)\right]$$
$$= \left(f_G \otimes_{u/d} h_G\right)(x) \cup \left(g_G \otimes_{u/d} h_G\right)(x)$$
$$= \left[\left(f_G \otimes_{u/d} h_G\right) \tilde{\cup} \left(g_G \otimes_{u/d} h_G\right)\right](x)$$

Thus, $(f_G \tilde{\cup} g_G) \otimes_{u/d} h_G = \left(f_G \otimes_{u/d} h_G\right) \tilde{\cup} \left(g_G \otimes_{u/d} h_G\right)$. □

Example 3.24. Consider the group G in Example 3.3. Let f_G, g_G, and h_G be three SSs over $U = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\rho, \{e, x, y\}), (\tau, \{x, yx\})\}, \quad g_G = \{(\rho, \{e, y\}), (\tau, \{e, yx\})\}, \text{ and } h_G = \{(\rho, \{e, yx\}), (\tau, \{y\})\}$$

Since $f_G \otimes_{u/d} h_G = \{(\rho, \{e, x\}), (\tau, \{x, y, yx\})\}$ and $g_G \otimes_{u/d} h_G = \{(\rho, \{e\}), (\tau, \{e, y, yx\})\}$, then

$$\left(f_G \otimes_{u/d} h_G\right) \tilde{\cup} \left(g_G \otimes_{u/d} h_G\right) = \{(\rho, \{e, x\}), (\tau, U)\}$$

Moreover, since $f_G \tilde{\cup} g_G = \{(\rho, \{e, x, y\}), (\tau, \{e, x, yx\})\}$, then

$$(f_G \tilde{\cup} g_G) \otimes_{u/d} h_G = \{(\rho, \{e, x\}), (\tau, U)\}$$

Thus, $(f_G \tilde{\cup} g_G) \otimes_{u/d} h_G = \left(f_G \otimes_{u/d} h_G\right) \tilde{\cup} \left(g_G \otimes_{u/d} h_G\right)$. □

Proposition 3.25. The soft union-difference product does not distribute over the union operation of SSs from the left side.

PROOF. Let f_G, g_G, and h_G be three SSs in Example 3.24. Since $f_G \otimes_{u/d} g_G = \{(\rho, \{x, y, yx\}), (\tau, \{x\})\}$ and $f_G \otimes_{u/d} h_G = \{(\rho, \{e, x\}), (\tau, \{x, y, yx\})\}$, then

$$\left(f_G \otimes_{u/d} g_G\right) \tilde{\cup} \left(f_G \otimes_{u/d} h_G\right) = \{(\rho, U), (\tau, \{x, y, yx\})\}$$

Moreover, since $g_G \tilde{\cup} h_G = \{(\rho, \{e, y, yx\}), (\tau, \{e, y, yx\})\}$, then

$$f_G \otimes_{u/d} (g_G \tilde{\cup} h_G) = \{(\rho, \{x\}), (\tau, \{x\})\}$$

Thus, $f_G \otimes_{u/d} (g_G \tilde{\cup} h_G) \neq \left(f_G \otimes_{u/d} g_G\right) \tilde{\cup} \left(f_G \otimes_{u/d} h_G\right)$. □

Remark 3.26. The soft union-difference product does not distribute over the union operation of SSs.

CONCLUSION

This study commences with the formal introduction of a novel product of soft sets, termed the "soft union-difference product", wherein the parameter space is endowed with a group structure. Building upon this foundational construct, we engage in a comprehensive algebraic analysis of the product, focusing particularly on its interaction with diverse taxonomies of soft subsets and generalized notions of soft equality. The development and systematic examination of such binary operations within a rigorously defined algebraic universe constitute a foundational pillar of abstract algebra. Specifically, the verification of core algebraic properties—including closure, associativity, commutativity, idempotency, and the existence (or absence) of absorbing element—enables the precise classification of the resultant algebraic system within the established algebraic taxonomy. Furthermore, the exploration of distributive laws and their compatibility over operations yields critical insights into the internal consistency and expressive algebraic power of the framework. The structural findings obtained herein not only illuminate the underlying mathematical regularities of the proposed operations but also demonstrate their potential to generalize and extend classical algebraic systems, offering new avenues for addressing unresolved problems in soft algebra. In this regard, the theoretical framework advanced in this work fills notable lacunae in the existing literature and lays a rigorous foundation for the emergence of a novel research direction in soft group theory predicated upon the proposed product. Prospective investigations may focus on the construction of additional soft algebraic operations and the elaboration of more nuanced equality frameworks, both of which are anticipated to significantly enrich the theoretical development and practical applicability of soft set theory within algebraic, computational, and decision-theoretic contexts.

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